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EARTHSYSTEM SCIENCES AWARENESS AND RESEARCH ASSOCIATION (REGD.8667/2000) (ESWARA)

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31st

Lecture of the Series in

ANCIENT INDIAN SCIENCES

Paratattva Ganita Darshanam

PARATATTVA GANITA DARSHANAM

(PRINCIPLES OF TRANSCENDENTAL PHILOSOPHY OF MATHEMATICAL TRUTH)

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The first of these two aphorisms discloses a universal principle of profound significance. It states that there are four means of grasping metaphysical truths: Word (*PADA*), Mystical letter (*BEEJA*), Number (*SAMKHYA*) and Line (*REKHA*) and that each succeeding one is superior to the former. Therefore, of these four means, the one that depends on Line (Geometry) is the best.

The second of the above two aphorisms states that *VRUTTAM* - a Circle represents *ESWARA* - translated as Godhead. Normally, the word *VRUTTAM* denotes the figure that we know as a Circle, which for ordinary purposes of study is the same as any other geometrical object. How does a Circle become qualified to represent God? Sri Jagadguru□□ Kalyanananda Bharati gave cogent arguments based on Upanishads to justify the statement that Circle □represents *ESWARA.*

Before we understand this statement, we must stress on a special feature of a Circle. And that is “a Circle represents roots of unity”. Further we are taught that √1, the square root of 1, has two values, 1 and also □1. Why should we accept both *positive one* and *negative one* as correct answers? Answers to both these questions are given through the 108 aphorisms in ***Paratattva Ganita Darshanam*.**

On the basis of mathematical truths, we can ponder on the metaphysical implication of the second aphorism, namely, *VRUTTAMEESWARAH*. If any small part of a circle is equal to the whole circle, is it not true that every part of *ESWARA* □□is indeed *ESWARA*□? In fact the message of *ADVAITA VEDANTA* is precisely this truth, namely *BRAHMAN* □is indivisible (*AKHANDAM*). The path to comprehend the answers to these questions is in the domain of complex numbers, familiar to mathematicians, which are nothing but points on circle of radius 1.

Thus, *Purna mimamsa darshanam* has enlarged the meaning of circle and thereby established a new way to negotiate through the complex concepts of *JIVA, ESWARA, AVIDYA* and MITHYA, and finally to comprehend the philosophy of *VEDANTA*.

In *Paratattva Ganita Darshanam*, the subject is presented in the form of a dialogue between two imaginary characters – a Mathematician and a *Vedantin*. In the lecture some examples like *SRI CHAKRAM* are also discussed as to how geometrical imagery is used for metaphysical meditations by the students of spiritual pursuits.

INTRODUCTION

Before I enter into the main subject of my talk, I would like to make a few preliminary remarks. There are three pillars are called Philosophy, Religion and Science for civilized life. The triad of Philosophy, Religion and Science are not independent of our daily life. (See Fig. 1)

Now, the subject of Mathematics is such that it has both roots and ramifications in all the three disciplines, Philosophy, Religion and Science. It does not need to be told that Mathematics is the basis of modern science.

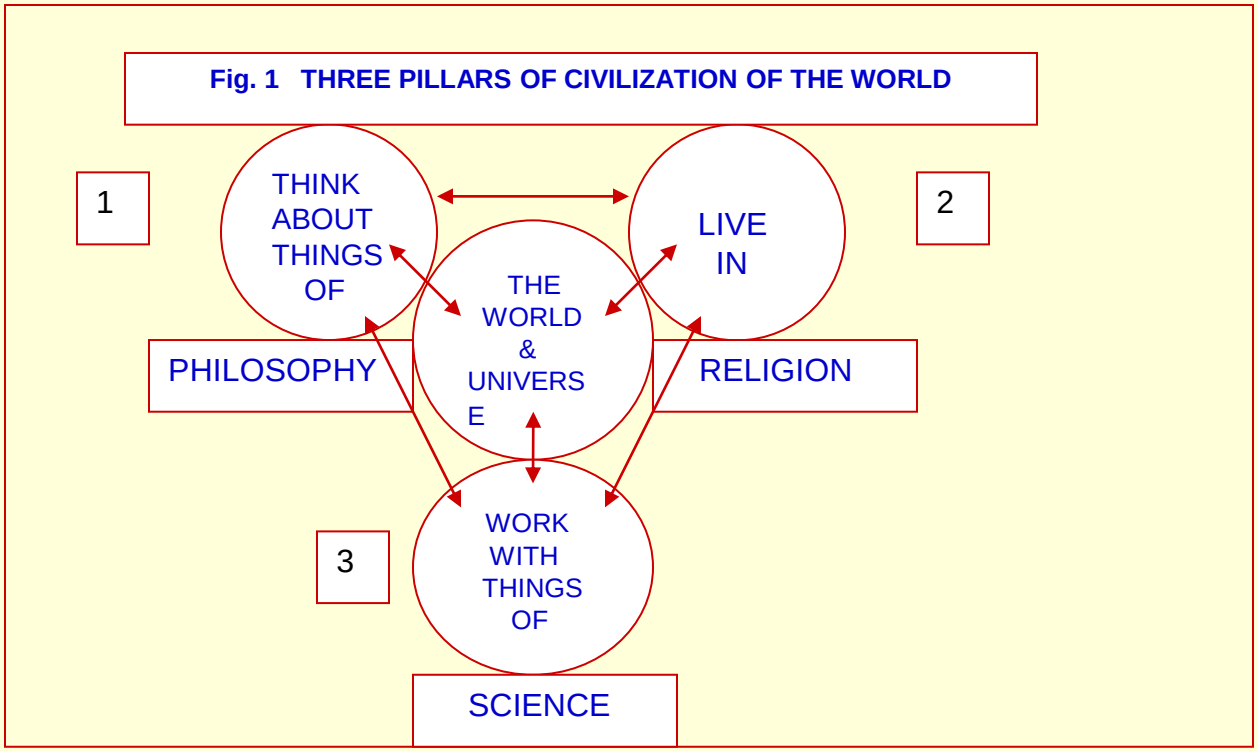


Fig. 1 THREE PILLARS OF CIVILIZATION OF THE WORLD

This talk reflects my faith stated in the following words:

1. If any mathematical truth exists it will be utilized by Nature.
2. This is justified in our study of the 'external' physical world.
3. Nature is Universal.
4. The 'inner', (metaphysical) world also is based on mathematical truths.
5. Vedanta gave universal truths, and Vedantic truths should also be mathematically true.
6. *Paratattvaganitadarsanam* finds evidence for this faith to be accepted as a valid metaphysical paradigm.

NATURE OF NATURAL NUMBERS

Numbers like 1, 2, 3 are called natural numbers. They involve confluence of TWO independent experiences in our consciousness. One, the abstract concept of a number and the association with **shapes in the world**.

Let us start with very elementary concepts, the concept of a number and pay spe cial attention to one operation, namely, multiplication of a number by itself. Let us look at the following Eq 1 $1 \times 1 = 1$,

It is pronounced as: “one multiplied by one equals one”. It is also written as $1^2 = 1$, and read as “one raised to the power of two equals one”. In either way, there is also a meaning to Eq.1 which is shown geometrically below: (Fig. 2)

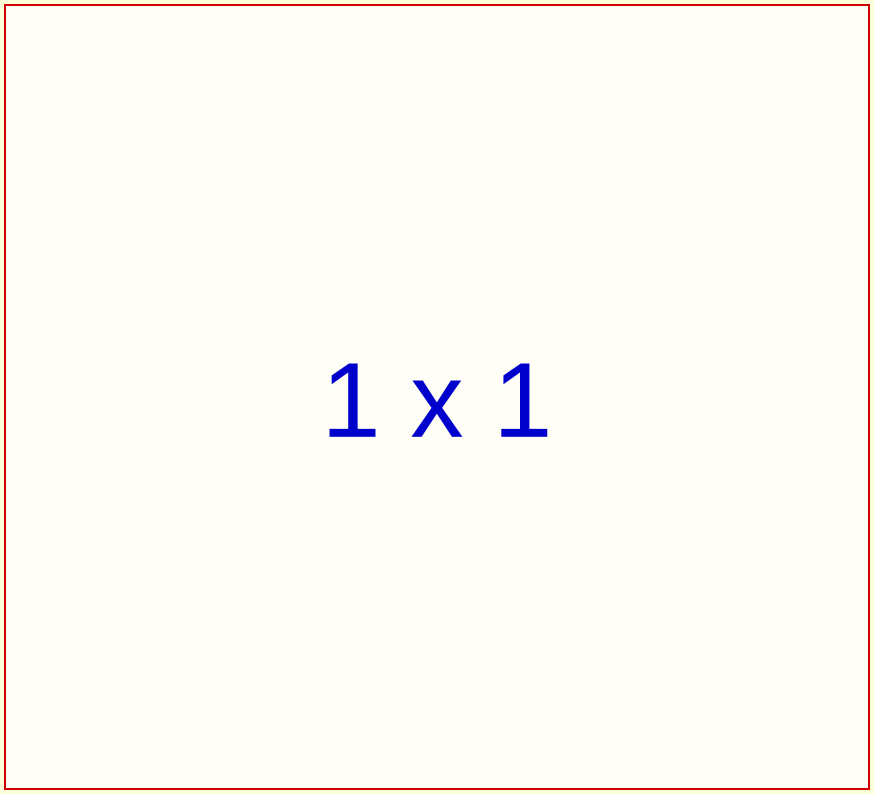


Fig 2: Multiplication increases the number dimensions.

Eq.2 $1 \times 1 \times 1 = 1$

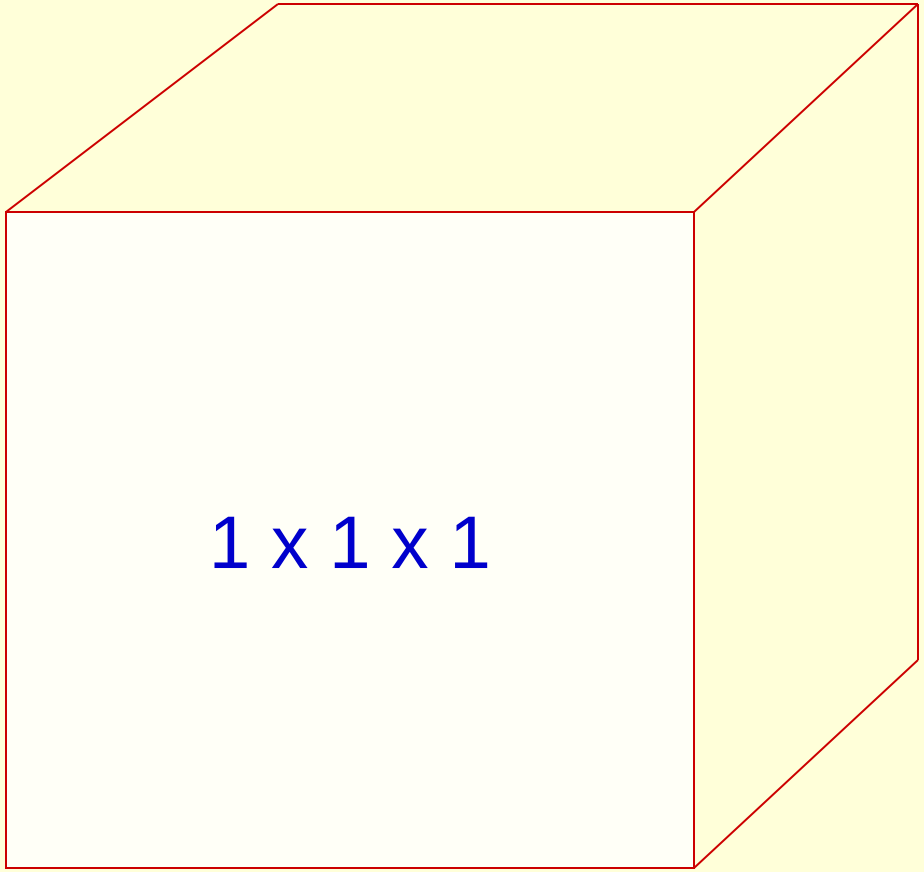


Fig:3 More multiplications create more dimensions

In the reverse operation of finding a square root of 1, we create a new number called negative number? What does this all mean for us?

A mathematician would say: we create an image space illustrated by means of diagram like (Fig 4)

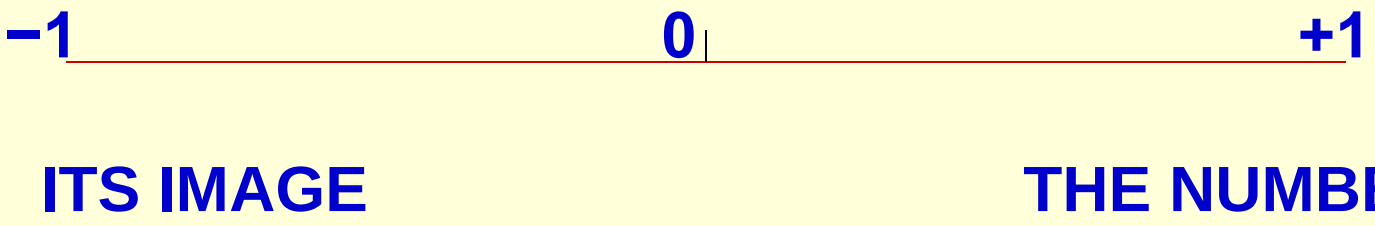
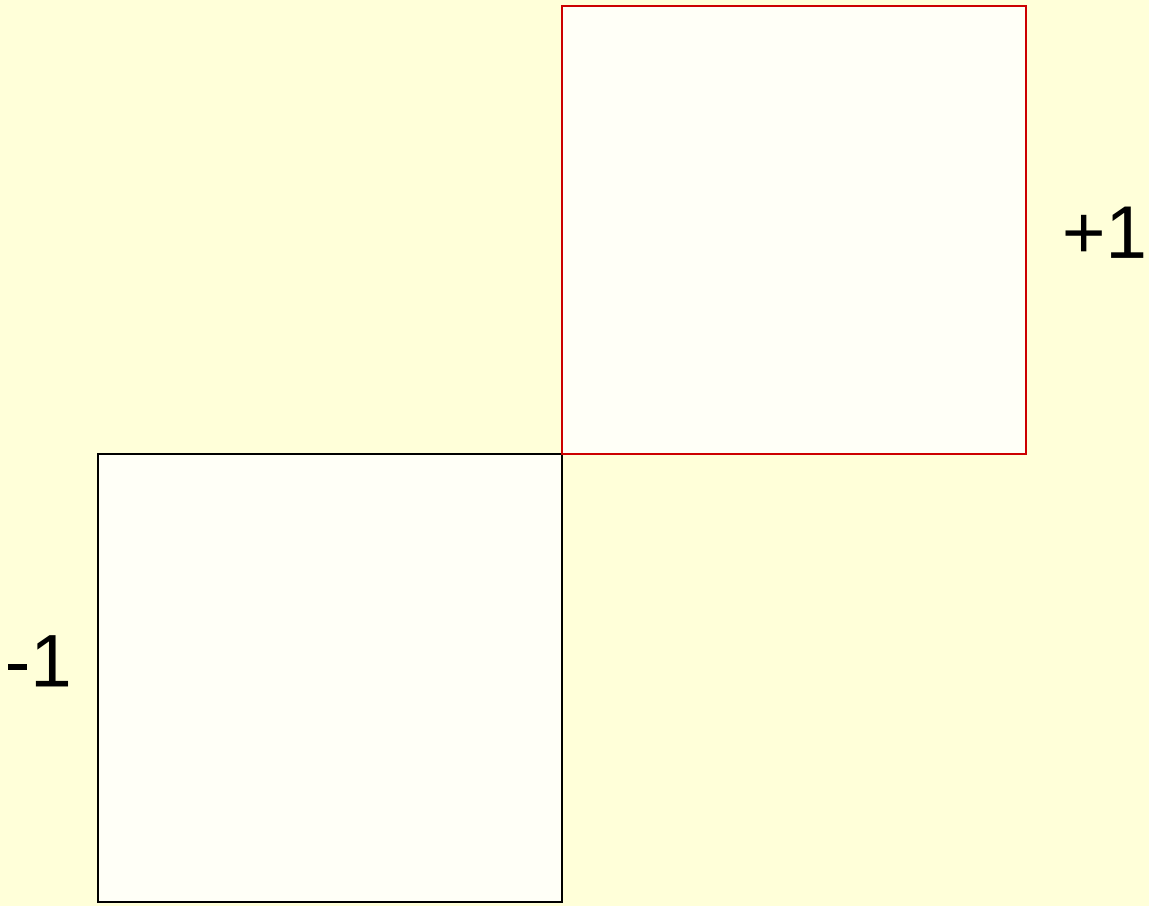


Fig : 4. Negative numbers are images of positive numbers separated by zero

An AXIOM is accepted that Zero, 0, separating the two regions exits which is its own image. But we shall see that this axiom can be amended. We have to accept concepts like tending to zero on the positive side and tending to zero on the negative side, such that the separation between the two is negligible and hence can stated to be Zero

The reflected numbers are put on the left side of “0” with a negative sign. The negative number also create spaces of higher dimensions like (Fig. 5)



Fig; 5. Negative numbers also create new space

DIFFICULTY ENCOUNTERED IN NUMBERS

Mathematicians encountered difficulty to comprehend the meaning of square root of a negative number. Such questions arose while solving algebraic equations. Even a quadratic equation like $x2 + x + 1 = 0$ leads to solutions $x = [-1 \pm \sqrt{(-3)}]/2$, which needs a square root of a negative number.

Unable to get a meaning of the such solutions, mathematicians decided to accept $\sqrt{(-1)}$ itself as a unit of what they called imaginary numbers. The unit of imaginary number is represented by the symbol, i , with the properties

$\mathbf{i = \sqrt{(-1)}}$	**Imaginary**
$\mathbf{i^2 = -1;}$	**Real**
$\mathbf{i^3 = -i;}$	**Imaginary**
$\mathbf{i^4 = 1}$	**Real**
$\mathbf{i^5 = i;}$	**Imaginary and so on**

TRANSCENDENTALISM IN NUMBERS

There are two numbers which are called transcendental numbers. This nomenclature came into mathematical literature because these numbers are proved to be neither rational nor irrational (algebraic) but they exit.(Ref; A Concise History of Mathematics. Dirk J. Struik, Dover Publ. 4th Revised Ed, 1987, page 180). One of these is the well known ratio of the circumference of a circle and its diameter, denoted by the Greek symbol π (Pi).

The other number, denoted by the English letter e, is the limit of a sequence of numbers defined as:

e = Limit of $[1 + (1/n)]^n$ as $n \rightarrow \infty$ (n = 1,2,3 .etc. tending to infinitely large values).

Let us look at the individual terms of the sequence.

We shall define them as:

$$\mathbf{A_n = \{1 + (1/n)\}^n\ ,\ n = 1, 2,.3\ldots..}$$

A_1, A_2 and A_3 etc. represent spaces of increasing dimensions Thus, A_1 represents one dimensional space - a line

A_n represents two dimensional space; a square

A_n represents three dimensional space; a cube

Therefore, as $n \rightarrow \infty$, $A_\infty \equiv e = (1 + 0_+)^\infty$ represents outer volume of infinite dimensional 'unit'.

Now we shall consider a variation and define a new sequence of numbers denoted by the symbol B_n , where

$$\mathbf{B_n = \{1 - (1/n)\}^n \, , \, n = 1,2,3\ldots \infty .}$$

Number of Dimensions	Shape Geometrical	$A_n = \{1+(1/n)\}^n$ "Outer Vol	"$B_n = \{1-(1/n)\}^n$ "Inner Vol"
1	Line	2	0
2	Square	2.25	0.25
3	Cube	2.37	0.296
4	4-Cube	2.44	0.3164

Following the arguments applicable to A∞, the number B∞ represents the inner volume of an infinite dimensioned unit.

It can be verified that B∞ is the inverse of A∞, and therefore, the product A∞ × B∞ = 1. Their numerical values are

A∞ = e = 2.718281827……,

B∞ = (1/e) = 0.367879 944….

THE FINAL CONCLUSION IS: "THE OUTER AND INNER VOLUMES OF INFINITE DIMENSIONAL UNIT CUBE ARE NOT EQUAL TO EACH OTHER, BUT ARE ACTUALLY INVERSES OF EACH OTHER".

Let us ponder a little at this stage about the philosophical implications of these numbers. The definitions of A_n and B_n are such that the size of the unit changes, either increases or decreases, with n, *but* by vanishingly small value for infinte number of dimensions.

The change in the size of the unit, inversely proportional to the number of dimensions, is BEYOND our experience in physical world. We know Relativity principle introduced dependence of units on state of rest or state of motion only.

However, one can read a meaning into this mathematical feature as follows:

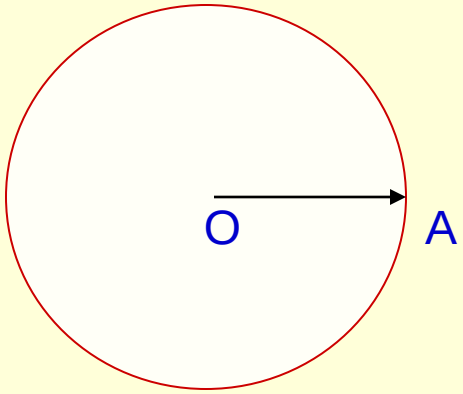
One may ask, “Is our experience of external and internal worlds analogous to that of A_∞ and B_∞ ? Do they represent property of a conscious individual?

Those of you who happened to read Dakshinamaurthy Stotram of Jagadguru Shankaracharya recall that

“Universe we are aware of, as existing outside of us, is like an image of a city seen in a mirror. But it is felt as if it is outside due to power of Maya”.

Are A_∞ and B_∞ like exterior and interior of a Self Conscious unit individual? Good Question to ponder on!

Known MATHEMATICS OF UNIT CIRCLE



Definitions

Fig: 6. Circle is locus of point ‘A’ keeping ‘O’ origin fixed.

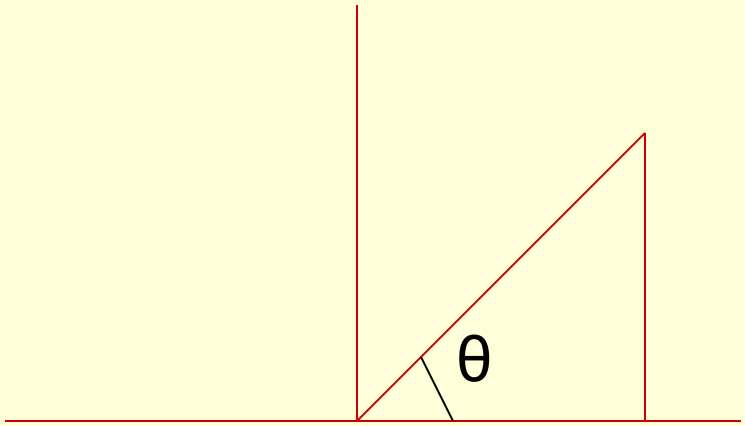


Fig: 7 Angle ± θ taken positive (anti-clock wise) or negative (clock-wise).

$Z = i\theta;$

By De Moivre (1806-1871) Theorem

$$A^{\infty}(z) = \exp(i\theta) = \cos\theta + i\sin\theta$$

$$B(-z) = \exp(-i\theta) = \cos\theta - i\sin\theta$$

Euler's equation (Leonhard Euler 1707-1783)

$$\exp(2n\pi i) = 1 \text{ , for all } n = 1,2,3,\text{etc}$$

PHILOSOPHICAL ISSUES OF ROOTS OF UNITY

They were recited in the Vedic hymns by the sages. (MAHANYASA)

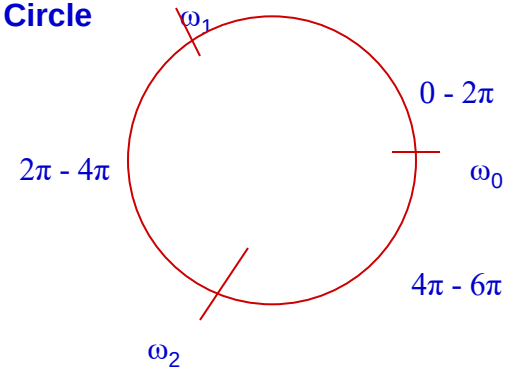
Also, we know that the Hindu system of numerals spread into the West through the Middle East only in the 13th century. (Ref Annemario Schimmel) **The phrase 'unit circle' came into existence in the 18th century.**

The mathematical truth that, exp (2nπ i) = 1 , for all n = 1,2,3, etc. has two meanings. One meaning is that a circle repeats itself as the angle changes with a period of 2π without any limit. The other meaning is that, as far as the property of circle is concerned, the interval 0 to 2π is the same as that 2π to 4π, 4π to 6π, 6π to 8π, and so on without any limit.

If so, we can also admit that there are infinitely large number of circles superposed on each other, though we are unable to see them individually. This argument allows the possibility that the interval 0 to 2π is not the same as that 2π to 4π, and so on. The nature of that possibility should be, however, specified. We shall in fact
see that such possibility exists.

Annemario Schimmel. *The Mystery of Numbers*. Oxford University Press. 1993.pp. 8 - 9.

Fig, 8 Location of CUBE ROOTS OF exp (2πi) On the circumference of Unit Circle



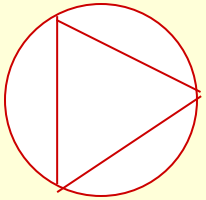


Fig 9 An Equilateral Triangle is inscribed in a Circle. It is called *Adhyasa*, Superposition in Vedanta. An Equilateral Triangle represents cube roots of unity. This feature forms the link between Vedanta and Geometry. Geometrically an Equilateral triangle inscribed in a circle has the same origin as the circumscribing circle

Let us consider the roots of unity. We know that

$\exp(2\pi i) = 1$, and therefore,

$\sqrt{\exp(2\pi i)} = \pm\exp(2\pi i/2)$

$= \pm \exp(\pi i) = \pm(-1)$.

Here, we get the proof for a negative root of 1.

For n =3, the cube roots are given by

$\omega 0 = 1$, $\omega 1 = \cos(2\pi/3) + i \sin(2\pi/3)$,

$\omega 2 = \cos(4\pi/3) + i \sin(4\pi/3)$.

In general, all the n roots of 1 will be vertices of regular polygon with n sides.

The geometrical importance of the roots of 1, is the fact that the arcs of the circle between successive roots, beginning from ω0 to ω1, ω1 to ω2, ω2 to ω3 , etc correspond to equal ranges of θ from 0 to 2π, 2π to 4π and 4π to 6π, etc. Therefore, we can conclude that roots of 1, which apparently divided the circumference into equal parts, actually produced as many multiples of 2π as the number of roots. These results hold good for any number of roots of 1, however large the number may be. The generation of innumerable intervals each of which is a copy of a given interval and superpose all of them on the given interval is a marvelous result. Simply, it means a large number of unit circles are reproduced on the unit circle. This brings to our mind the sentence of Lord Krishna

"*Mayi sarvamidam protam sutremaniganaviva*".

The cube roots of 1 have a special significance. They can represent a polygon with minimum number of sides inscribed in a circle.

If the roots are joined to the center of the circle by straight lines, they look like three strings connecting the triangle to the center. A string is called *guna* in Sanskrit. In metaphysics, it is stated that the Nature is composed of three qualities. The power underlying the universe is called Maya comprised of three *gunas*.

Lord Krishna states that this power is under His control. *Daivi hyesha gunamayi mama maya duratyaya mameva ye prapadyante mayametam taranti te*. Here again we see another feature of mathematics echoing metaphysical notions in the language of geometry.

INQUIRY INTO NATURE OF TIME

We recall that the unit circle is given by $|\exp(i\theta)| = 1$, and it is instructive to re-examine again the exponential functions defined in $A_\infty(i\theta) = \exp(i\theta)$ and $B_\infty(-i\theta) = \exp(-i\theta)$, where we see that increasing values of θ in A_∞ correspond to decreasing values of θ in B_∞. In our analysis, A_∞ and B_∞ correspond respectively to the external and internal volumes of infinite dimensional unit cube.

If the variable θ is understood to be an angle, we can interpret the change in the sign of θ as indicating a measurement of the angle either in clockwise direction, or in opposite direction. And so, we can accept this as a curious geometrical property which reflects difference between the internal and external sides of the infinite dimensional unit cube.

But, if we assume that θ corresponds to Time, we face some difficulties though we are familiar with periodic phenomenon of rising sun and setting sun. We do not know how to measure a decrease of Time. As far as we can imagine, Time seems to be unidirectional.

However, there is also another aspect of Time which we can take into consideration. We have some 'experience' of Time as past Time and Future Time. We can understand these two aspects in terms of the properties of A_∞ and B_∞. Let us remember that A_∞ and B_∞ correspond respectively to outer and inner volumes of infinite dimensional unit cube. If so, there (seems to be) is a correspondence of the outer and the inner regions to the future and past nature of Time by identifying θ as Time, (leaving aside the debate about the nature of time in philosophy).

Ruth Reyna *Metaphysics of Time in Indian Philosophy and its Relevance to Particle Science.* Article in *Time in Indian Philosophy*. Ed. Hari Shankar Prasad. Publ.Sri Satguru Publ. Delhi.1992. pp723-734.

Thus, the infinite dimensional unit cube qualifies itself to represent a Conscious Entity. For such entity, the exterior is the future of its interior is the past. Thus, the future and past are inseparably connected to the infinite dimensional unit cube, which is eternal and independent of Time.

If we follow the Aphorism “Vrittameeswara” , we arrive at the conclusion: FUTURE TIME AND PAST TIME ARE INSEPARABLY CONNECTED TO ETERNAL GOD AS HIS EXTERIOR AND INTERIOR. In this case, the unit circle OR the infinite dimensional is the ETERNAL WITNESS OF TIME. IT IS ITSELF TIMELESS.

We can further illustrate this concept utilizing the principle of invariance of mathematical truth with change of scale.

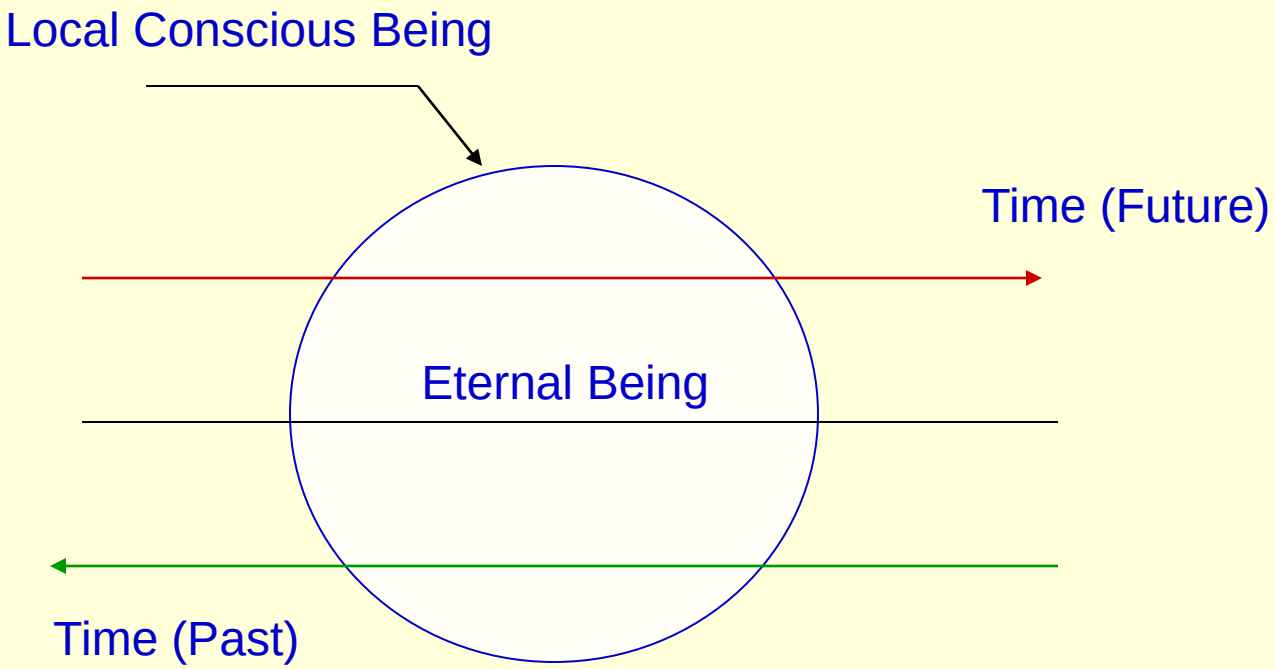


Fig: 10. Local Nature of Past and Future in relation to An Eternal Being

Implications of this result are interesting. When Lord Krishna declared that He knew all His previous incarnations, we cannot dismiss it as a fiction.

"You reap as you sow".

As conscious individuals, we are aware, in a limited manner, of our future and past, though restricted by higher powers to the period of our existence.

It may be realized at this stage that our concept of time is intimately dependent on the existence of $i\theta$, the imaginary number. Time is a power controlling all the transient universe.

But it has its 'seat' on A∞ and B∞ Hence it is not absolute. But in its absence, i.e. in the absence of Time, there is no unit circle either.
What exists only is 'unity'.

That is the Absolute *EKAMEVA ADVITIYAM NEHA NANASTI KINCHANA*.
In other words, Time is Unit Circle and Unit Circle is Time. Time is the dynamic power. It is described as *Devatma Sakti, swagunaih nigudham.*

In summary, the mathematical constant ‘e’ represents Space of infinite dimensions, and iθ represents the infinite Time, both together make up the **unit circle.** The universe of multiplicity is represented by roots of 1. Taking a clue from our limited consciousness, we are able to get a hint that the **unit circle** represents the Eternal witness. Can we progress further to understand more of Vedanta?

INQUIRY INTO NATURE OF *MITHYA* AND *AVIDYA*- A Model

Consider again the cube roots of unity.

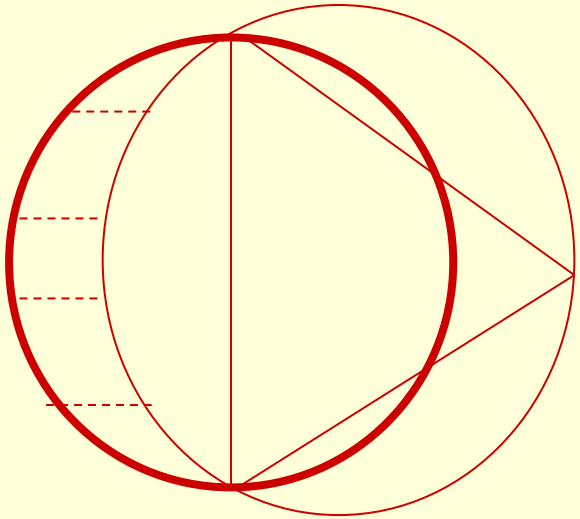


Fig. 11. Illustration of Avidya: A circle, in thin line, has an inscribed equilateral triangle. Taking one side as diameter, a new circle is drawn in thick line. The two circles make up a crescent-shaped figure called Lune outside the triangle. The crescent has no existence without the triangle. The triangle has no existence without the circle in thin line. Avidya is that state of knowledge of the crescent, unaware of the existence of the arc in thin line and the existence of an equilateral triangle. Knowledge of arc of thick line alone, without the knowledge of the triangle and the knowledge of circe in thin line, is designated as Avidya. Therefore, Avidya is neither pure ignorance nor pure illusion, but it is as incomplete knowledge. The crescent (Lune) represents the state of Jiva in Vedanta

Lack of awareness (called ignorance) of the basis can be interpreted as what is called *Avidya* in Sanskrit. It may also be interpreted as Incomplete Knowledge, because *Avidya* can be overcome by correct knowledge.

But as a matter of mathematical truth, the arcs, made by the cube roots of 1, are all identical to the unit circle. The crescent is formed jointly by the arc of unit circle and the secondary circle. The nature of *Avidya* is actually the unawareness of the presence of arc of the unit circle.

The nature of mathematics is not a fixed property. It changes with the way it originates and is used by a conscious individual, whose approach to mathematics is largely influenced by the world in which he/she lives. Though it is true that mathematics was initially a tool for material needs, its potential is not restricted to that activity only. ‘Mathematics has just as much narrative, purpose, and story telling in it as it has calculation and formulas. If we fail to see this and remain ignorant of mathematics but blindly reverent toward its techniques, we impoverish ourselves needlessly and empower others excessively.’ [1]

[1] John Allen Paulos. *Beyond Numeracy*. p. 177

SELF AND GODHEAD

Now we shall expound the (metaphysical) principles of (geometrical) line.

Circle (is) the prior form.

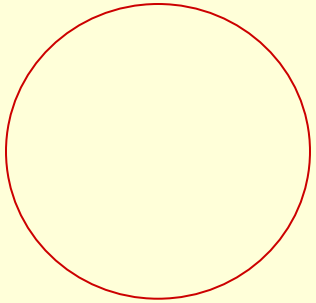


Fig. 1 The Circle (is) the prior form.

Diameter is the latter form.

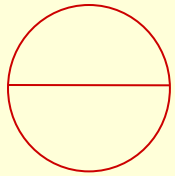


Fig. 2. The diameter is the latter form.
A pair of points (is their) junction.

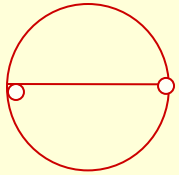


Fig. 3 A pair of points (is their) junction.

Infinitude (is their) connection.

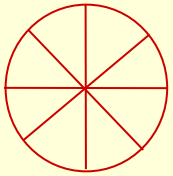
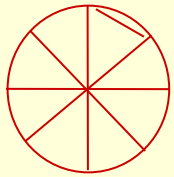


Fig.4 Infinitude (is their) connection

This is (with regard to) Godhead.

Therefore, (the aphorisms on) the notion of Self (are given separately

Diameter (is the) prior form



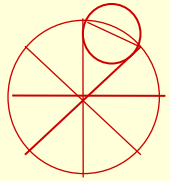


Fig.5 Diameter (is the) prior form.

Circle is the latter form.

A Pair of points (is) the junction.

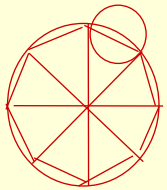


Fig. 7 A pair of points (is) the junction.
Infinitude (is their) connection.

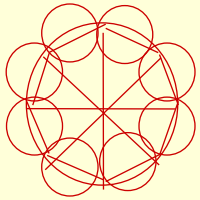
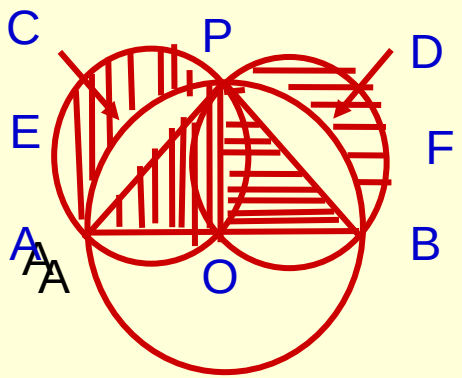


Fig.8 Infinitude (is their) connection.

This (is regarding) the notion of the (individual) Soul.

Thus, (we have expounded) the metaphysical principles of (the geometrical) line.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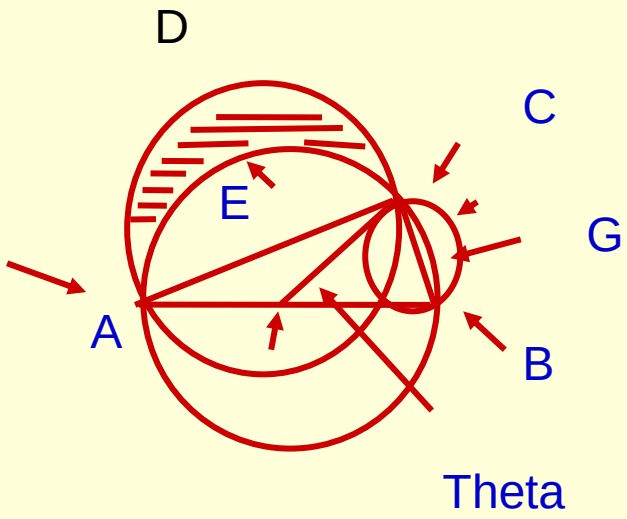
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Fig. 11. Illustration of Avidya: A circle, in thin line, has an inscribed equilateral triangle. Taking one side as diameter, a new circle is drawn in thick line. The two circles make up a crescent-shaped figure called Lune outside the triangle. The crescent has no existence without the triangle. The triangle has no existence without the circle in thin line. Avidya is that state of knowledge of the crescent, unaware of the existence of the arc in thin line and the existence of an equilateral triangle. Knowledge of arc of thick line alone, without the knowledge of the triangle and the knowledge of circe in thin line, is designated as Avidya. Therefore, Avidya is neither pure ignorance nor pure illusion, but it is as incomplete knowledge. The crescent (Lune) represents the state of Jiva in Vedanta

The nature of mathematics is not a fixed property. It changes with the way it originates and is used by a conscious individual, whose approach to mathematics is largely influenced by the world in which he/she lives. Though it is true that mathematics was initially a tool for material needs, its potential is not restricted to that activity only. ‘Mathematics has just as much narrative, purpose, and story telling in it as it has calculation and formulas. If we fail to see this and remain ignorant of mathematics but blindly reverent toward its techniques, we impoverish ourselves needlessly and empower others excessively.’ [1]

[1] John Allen Paulos. *Beyond Numeracy*. p. 177

A Theorem on a pair of unequal lunes and the Metaphysical Pair of Birds in Vedanta

A lune is a geometrical figure formed on a plane by two intersecting arcs of circles or on a sphere by two great circles. Hippocrates of Chios (c. 430 B.C.) discovered that a lune bounded by an arc of $\pi/2$ radians and by a semi-circle upon its chord is equal in area to the triangle formed by the corresponding chord with the center as the apex. [70] This is illustrated in the Fig. 1.

Fig. 1. The discovery of Hippocrates of Chio.

In Fig. 1, O is the center of the circle and AOB is the diameter, and OP is perpendicular to AOB; AP and PB are chords which subtend an angle $\pi/2$ at the center and the $\angle POB = \angle POA = \pi/2$. Therefore, the small arcs ACP and PDB subtend an angle $\pi/2$ radians at the center. With AP and PB as diameters two circles are drawn forming two lunes shown by the shaded areas by lines drawn along north-south and east-west directions. The triangles with AOP and POB are similarly shaded.

The area of the lune formed by the small arc ACP is equal to the area of the triangle AOP. Similarly the same result holds for the other lune and corresponding triangle. This is the theorem of Hippocrates of Chio.

The nature of the relation between the lune and the triangle constructed as above when the angles subtended at the center is not equal to $\pi/2$ seems to be very complicated but a simple result can be deduced for a pair of lunes that subtend supplementary angles at the center.

The Fig. 2 shows the construction of two lunes that subtend supplementary angles at the center.

Let AOB be the diameter of the circle with the center at O. The arc AECFB is one semi-circle. Join AC and CB to form the triangle ACB. Let the angle COB be equal to θ, and therefore the angle COA is equal to $\pi - \theta$. With AC and CB as diameters draw the semi-circles ADC and CGB respectively to form pair of lunes ADCEA and CGBFC. See Fig 2.

Fig 2. Construction of a pair of unequal lunes.
(Angle, θ, measured anticlockwise)

Now, we prove the following theorem: The sum of the areas of lunes ADCEA and CGBFC is equal to sin θ, which is the area of the triangle ACB.

Proof: Let us first derive an expression for the area of lune bounded by ADCEA. Assume for simplicity that OA = 1. We know from the rules of Euclidean geometry that the length of the chord AC is 2 [sin $\{(\pi-\theta)/2\}$].

Therefore the area S1 of the semi-circle ADC is, noting OA=1,

$$S1 = (\pi/2)\ \text{sin2}\ [\{(\pi-\theta)/2\}]$$

Further, the area S2 of the Fig. enclosed by the arc AEC and the radii OA and OC is

$$S2 = (\pi-\theta)/2$$

The area S3 of the triangle AOC is

$$S3 = \sin [\{(\pi-\theta)/2\}] \times \cos [\{(\pi-\theta)/2\}]$$

$$= (1/2) \sin[2\{(\pi-\theta)/2\}] = (1/2) \sin [(\pi-\theta)]$$

$$= (1/2) \sin \theta.$$

Therefore the area S4 of the lune ADCEA is given by

$$S4 \equiv S1 - (S2 - S3)$$

$= (\pi/2) \sin 2 [\{(\pi - \theta)/2\}] + (1/2) \sin \theta - (\pi - \theta)/2$

$= (\pi/2) \cos 2 [\theta/2] + (1/2) \sin \theta - (\pi - \theta)/2.$

The area S5 of the lune CGBFC is obtained from the above equation for S4 by changing θ to $\pi - \theta$.

Therefore

$S5 = (\pi/2) \sin 2 [\theta/2] + (1/2) \sin \theta - \theta/2.$

And, finally the S6, the area of the sum of the two lunes is given by

$S6 = S4 + S5 = \sin \theta,$

which is the area of the triangle ACB since the area of triangle COB is equal to that of triangle AOC because both have equal bases and heights.

(Q.E.D).

It is interesting to examine the ratio R of the area of lune ADCEA and that of triangle OAC as a function of θ. It can be verified that R is given by

$$R = 1 + (\pi/2) \cot(\theta/2) - (\pi - \theta)/\sin\theta$$

When $\theta = \pi/2$, we obtain R = 1 which agrees with the result of Hippocrates. It can be verified that R tends to 0 or 2 according as θ tends to π or 0. This shows that the area of lune ADCEA becomes equal to the area of the triangle ACB while the area of the lune CGBFC tends to vanish as θ tends to vanish or OC tends to coincide with OB.

This result has interesting metaphysical features. Both the lunes have sides of the same triangle as their support. According to the principle of the supporter and the supported, the triangle ABC is the support and the lunes are the supported. Though both the lunes are on the same footing, they are not equal except when $\theta = \pi/2$. The lune corresponding to $\theta < \pi/2$ has a 'companion' lune with $\theta > \pi/2$. This reminds us of the Pair of Birds well-known from the III. 1.1 and III.1.2) [71]

The verses quoted above become meaningful if they are seen in the light of the geometrical imagery. The details will be discussed in the main text.

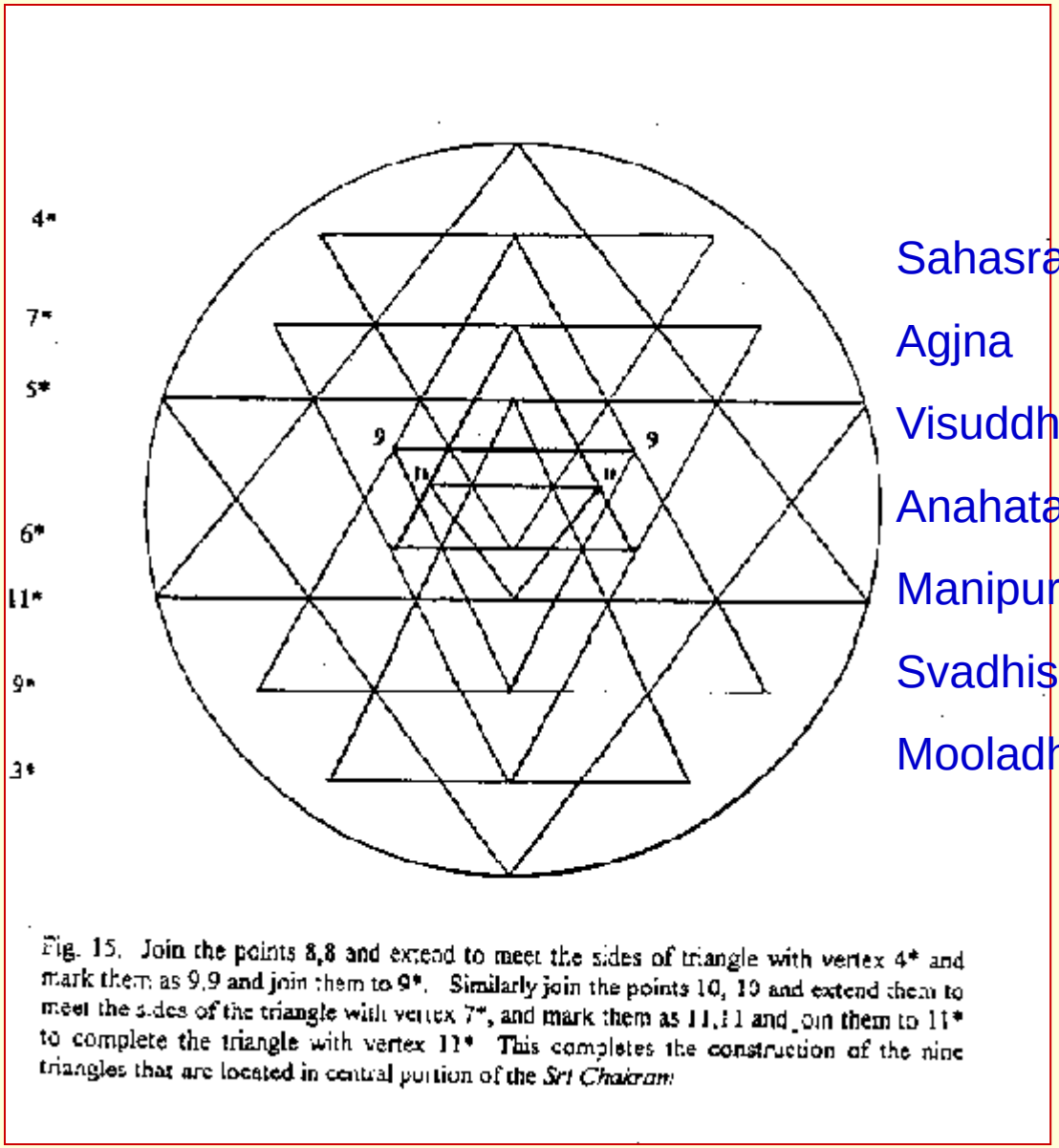

Fig. 15. Join the points 8,8 and extend to meet the sides of triangle with vertex 4* and mark them as 9,9 and join them to 9*. Similarly join the points 10, 10 and extend them to meet the sides of the triangle with vertex 7*, and mark them as 11,11 and join them to 11* to complete the triangle with vertex 11* This completes the construction of the nine triangles that are located in central portion of the *Sri Chakram*